DEPARTMENT OF THE ARMY
U.S. Army Corps of Engineers
Washington, DC 20314-1000

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Technical Letter
No. 1110-2-336

31 July 1994

Engineering and Design
OPERATION OF RESERVOIR SYSTEMS

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1. Purpose

This engineer technical letter (ETL) presents field water-control managers a new tool for developing and evaluating reservoir system water control plans. This ETL expands the information on water-control analysis techniques presented in Chapter 6 of EM 1110-2-3600. A new software optimization package for reservoir system analysis is presented.

2. Applicability

This ETL applies to all HQUSACE elements, major subordinate commands, districts, laboratories, and field operating activities (FOA) where it is necessary to perform reservoir system analysis. The primary expected application is for the determination of reservoir water control plans.

3. References

References and additional sources of information are listed in Appendix A.

4. Reservoir Analysis Models

A brief description of computer programs for reservoir analysis is provided in Appendix C of EM 1110-2-1701. The models are listed as either flow-duration or sequential streamflow, plus one hybrid. A Hydrologic Engineering Center model review (HEC 1991a) provides a "...literature-review-based assessment of the state-of-the-art of modeling and analysis approaches for evaluating multiple-purpose reservoir system operations." Models, in this report, are categorized as descriptive simulation, prescriptive optimization, or hybrid simulation and optimization models. To date, most of the Corps' analyses have been performed with sequential streamflow (descriptive simulation) using one of the generalized models, or a specialized model for the system.

5. Descriptive Reservoir Simulation

a. Methods. Reservoir simulation is performed by repeatedly solving the storage equation for a reservoir *(inflow minus outflow equals change in storage)*. The simulation is descriptive because the system and its output requirements are all specified, e.g., the sequence of flow data, storage allocation, and project demands, priorities, and constraints. Given this description of the system, the output is the reservoir releases and the resulting reservoir storage and downstream flows. Chapter 5 of EM 1110-2-1701 provides a complete description of sequential streamflow routing for hydroelectric power. The same concepts and general procedures apply to other water conservation purposes. Reservoir simulation for flood control is presented in IHD Volume 7 (HEC 1976). The procedures outlined in Volume 7 were incorporated into the HEC-5 Simulation of Flood Control and Conservation Systems (HEC 1982). As mentioned in paragraph 4, the capabilities of HEC-5 and other computer models are summarized in EM 1110-2-1701.

b. Application. Reservoir simulation is a powerful tool because it allows the modeler to utilize the level of detail, and the available data, required to meet the objective(s) of the analysis. Sequential reservoir analysis can consider almost any physical process that could affect the reservoir inflow, outflow, and release determination. Typically, processes are defined as a functional relationship, or as a period-by-period input. The application approach is "case study," in that an operation policy, flow sequence, and system demands are specified and the simulation is performed to determine the result. Different "cases" are analyzed by changing the operation policy, demands, or other aspect, and running the simulation again. The simulation is *descriptive* of the expected reservoir operation, given the specified scenario.

c. Limitations. A disadvantage of this approach is the difficulty analyzing the large number of alternatives possible with a multiple-purpose, multiple-reservoir system. Additionally, most sequential simulations treat specified targets and demands as absolutes. The reservoirs release water to meet the specified demands as long as there is available water in the allocated water supply storage. The flood-control space is typically used in the same fashion, i.e., store floodwater if there is space available. While it is fairly easy to compare absolutes (e.g., flood control vs. conservation storage) it is harder to evaluate the trade-offs in operation policy.

6. Prescriptive Reservoir Optimization

a. Prescriptive vs. descriptive. A *descriptive* tool answers the question "How would the system perform if we followed this policy or set of priorities?" (HEC 1992a). A *prescriptive* tool is used to answer the question "How should we operate the system if we accept this definition of the goals of, and constraints on system operation?" (ibid). A prescriptive tool generates iteratively the alternative policies to be considered and evaluates the feasibility of each with a built-in simulation model. It quantifies the efficiency of each feasible alternative using a formal definition of operation goals and objectives. Finally, after evaluating all alternatives, it identifies the best policy. Examples of prescriptive tools are linear-programming, nonlinear-programming, and dynamic-programming models.

b. HEC-Prescriptive Reservoir Model. HEC has developed and applied a Prescriptive Reservoir Model (HEC-PRM) to analyze the operation of the Missouri and Columbia River Systems (*see*: HEC 1991c, 1991d, 1992a, and 1993 *in paragraph 3, Appendix A*). In HEC-PRM, a reservoir system is represented as a network of arcs connected at nodes. The arcs represent any facility for the transfer of water, both in space and time. The nodes represent reservoirs or other locations where flow is required or evaluated. The value of water in the system is defined in terms of penalties for flow, or water in storage, being too high or too low. The allocation of water in space and time is treated as a Minimum-cost Network-flow Problem. A more complete HEC-PRM description is provided in Appendix B.

c. Penalty functions. The penalty functions for flow, or water in storage, are developed for each project purpose, at each location, for each month of the year. The single-purpose penalty functions are then combined into composite functions at each location for each month of the year. The resulting combined functions are then edited, or smoothed, to yield a piecewise-linear convex function for the network solution. The requirements and general procedure are described in paragraph 5, Appendix B.

7. Data Requirements for HEC-PRM

There are three sets of data required for the model: hydrologic data, penalty data, and reservoir system data (HEC 1993, paragraph 2, Appendix A). Additional hydropower data may be required for reservoirs with significant pool variation.

a. Hydrologic data. The program requires flow data in the same units as storage data. Applications to date have used thousands of acre-feet per month. Flow data are input for upstream reservoir inflow and the incremental area inflow for downstream locations. While HEC-PRM allows the user to define a second hydrograph to define fixed depletions and an evaporation rate per month, the applications to date have made these types of adjustments to the input flow data. The data are read from an HEC-DSS file (HEC 1987b). The HEC-DSS utility programs provide for importing data from other files, and for manipulating the data to develop the required input to HEC-PRM.

b. Penalty data. This is the critical input for the HEC-PRM program. The program goal is to determine the reservoir operation that minimizes the total penalty for the simulation period. Obviously, there must be acceptance of the penalty values determined for each purpose in order to accept the resulting reservoir operation. The program summary (Appendix B) provides a description of typical penalty functions. For each node and month, the individual penalty functions are summed into composite penalty functions and they are stored in HEC-DSS. A utility program has been developed to read the composite functions from an HEC-DSS file and develop the convex, piecewise linear representation required by PRM. The utility provides a graphical display of the original and edited function, and allows the user to select the number of linear elements and to adjust the function values. An error value is displayed showing the relative mean deviation of the computed function from the input composite function.

c. Reservoir system data. The reservoir system data define the reservoir storages, the downstream connectivity, and the record path names to read the flow and penalty data from HEC-DSS. Minimum and maximum constraints on reservoir storage and channel flow are also defined. There is no routing in the model, so time-steps must be large enough for the flow to pass through the system within one time-step. Monthly data have been used. The reservoir data are defined in an ASCII file, with an input structure similar to other HEC programs. The HEC-PRM User's Manual (HEC 1993) provides the input requirements and formats.

d. Reservoir power data. When power reservoirs have a significant pool variation, hydropower capability and required hydropower releases depend on reservoir pool level. For specified reservoir storage values, a family of power capacity and hydropower penalty curves can be defined and stored in an HEC-DSS file. The hydropower penalty is assigned to a hydro-release link only. HEC-PRM can cycle to adjust storage, based on estimated outflow, in order to obtain the appropriate capacity and penalty values. This approach was used in the Columbia River System model (HEC 1993, paragraph 3, Appendix A).

8. HEC-PRM Output

a. Output tables. The primary output for the model is reservoir storage and outflow, and the total flow at each node, all written to an HEC-DSS file. Additionally, the program can compute reservoir elevation and energy production, if the conversion data are provided. A total system penalty value is computed based on the edited composite penalty functions; however the post-processor also computes the individual penalties for every purpose and sums them for each location. The time-series penalty data are also written to the output DSS file. A utility program has been developed to produce output tables from the results written to a DSS file. The utility can produce a variety of pre-defined and user-defined output tables of reservoir and node data, over time, or annual summaries. The utility also provides a graphical display of time-series data stored in the DSS file. Additionally, data written to HEC-DSS can be displayed with computer program DSPLAY (HEC 1987b).

b. Output interpretation. While standard tables of information on reservoir operation and the resulting penalty values can be produced, *the basis for reservoir release decisions must be inferred from the operation.* The operation results produce the minimum total penalty. The question is "How do we operate the real system to achieve the maximum benefit?" The HEC-PRM results must be analyzed and interpreted to formulate an operation plan. The output utility can provide duration and statistical data and plots, for any specified period and season, to facilitate output analysis. The derived operation plan could then be "tested" with a more detailed reservoir simulation. At this time, there has been limited application and only one systematic processing of preliminary output has been documented (HEC 1992b).

9. HEC-PRM Limitations

a. New software. Because this is a new reservoir system program, there has been limited application. The program is available and producing reasonable results, judging by the comparison between MRD simulation model and HEC-PRM results for a "normal" flow period. However, the results were not similar for a critical period analysis.

b. Limited simulation capabilities. As with most optimization models, the simulation aspects of the model are limited. Continuity is maintained. Specified maximum and minimum storage and flow constraints are observed. Hydropower capability can reflect variation in pool level. There is no routing in the simulation; therefore, applications are limited to large time-steps, e.g., monthly data.

c. Inferred operation policy. The basis for the period-by-period operation is hidden from the modeler. The results must be analyzed to develop insight into the operation policy that would produce similar results. The limited experience to date makes it difficult to specify the analysis strategy to use to develop an operation plan. Ongoing applications and analyses will give better insight in the near future.

FOR THE DIRECTOR OF CIVIL WORKS:

2 Appendices
APP A - References
APP B - Prescriptive Reservoir Model

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Chief, Engineering Division
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APPENDIX A: REFERENCES

A-1. Guidance and Regulations

ER 1105-2-100
Guidance for Conducting Civil Works Planning Studies

ER 1110-2-240
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APPENDIX B: PRESCRIPTIVE RESERVOIR MODEL*

B-1. Summary

a. The reservoir system water control problem is addressed as a problem of optimal allocation of available water. A prescriptive model is developed to solve this problem. The model identifies the allocation that minimizes poor performance for all defined system purposes. Performance is measured with analyst-provided penalty functions of flow or storage or both.

b. To determine the optimal water allocation, the physical system is represented as a network, and the operating problem is formulated as a minimum-cost network flow problem. The objective function of this network problem is the sum of convex, piecewise-linear approximations of the penalty functions. A network solver is used to determine the optimal allocation of water within the system. The results of the solver are processed to report and display reservoir releases, storage volumes, channel flows, and other pertinent variables.

c. To the extent possible, the software to implement the model is general purpose. Accordingly, the software includes the following model-building components:

- Inflow link.
- Initial-storage link.
- Reservoir-storage link.
- Final-storage link.
- Simple reservoir-release link.
- Hydropower reservoir-release link.
- Diversion link.
- Channel-flow link.
- Node.

An analyst can specify the characteristics and the configuration of these components to represent any system.

d. References cited in this appendix are listed in paragraph B-6. Special terms used herein are explained in paragraph B-7.

B-2. Problem Statement

a. The problem addressed by the system model is identification of an optimal long-term water control plan for the reservoirs of that system. This plan will identify the priorities to be assigned to conflicting objectives of operation. For example, the plan will identify whether and how much water should be released from a system reservoir if a demand exists for downstream flow for wildlife protection and a conflicting demand exists for continued storage of the water for reservoir recreation.

b. The model can quantify system performance for various purposes in multi-objective terms. The economic cost of operation is considered. Also, the social and environmental costs are considered. These costs are expressed in commensurate terms to permit display of trade-offs in operation for various purposes.

c. Constraints on the physical system are included. For example, the outlet capacity of the reservoirs can be modeled explicitly. However, inviolable constraints on system operation will be used frugally. This will avoid the problem described by Hitch and McKean (1960) when they wrote "...casually selected or arbitrary constraints can easily increase system cost or degrade system performance manyfold, and lead to solutions that would be unacceptable to the person who set the constraints in the first place." Instead, operation limitations are imposed through value functions. This will permit clear evaluation of the impacts of limitations. For example, instead of specifying maximum flow requirements for flood control, the system model will

* Adapted from: "Missouri River System Analysis Model - Phase I," Appendix C, February 1991.

represent this requirement through high costs of failure to meet the requirement.

B-3. Proposed Solution

The proposed solution considers the reservoir water control planning problem as a problem of optimal allocation of available water. The proposed solution to this water allocation problem is as follows:

- Represent the physical system as a network.

- Formulate the allocation problem as a minimum-cost network flow problem.

- Develop an objective function that represents desirable operation.

- Solve the network problem with a commercial solver.

- Process the network results to define, in convenient terms, system operation.

a. Represent system as a network.

(1) For solution of the water allocation problem, the reservoir system is represented as a network. A network is a set of arcs that are connected at nodes. The arcs represent any facilities for transfer of water between two points in space or time. For example, a natural channel transfers water between two points in space and is represented by an arc, as illustrated in Figure B-1. A reservoir transfers water between two points in time by an arc, as illustrated in Figure B-2.

(2) Network arcs intersect at nodes. The nodes may represent actual river or channel junctions, gage sites, monitoring sites, reservoirs, or water-demand sites. Flow is conserved at each node: the total volume of water in arcs originating at any node equals the total volume of water in arcs terminating at that node.

(3) Figure B-1 illustrates a simple single-period network representation. Node 3 represents a reservoir. Node 4 represents a downstream demand point. Two additional nodes with associated arcs are included to account completely for all water entering and leaving the system. Node 1 is the source node, a hypothetical node that provides all water for the system. Node 2 is the sink node, a hypothetical node to which all water from the system returns. The arc from node 1 to node 3 represents the reservoir inflow. The arcs shown as dashed lines represent the beginning-of-period (BOP) and end-of-period (EOP) storage in the reservoir. The BOP storage volume flows into the network from the source node. The EOP volume flows from the network back to the sink node. The arc from node 3 to node 4 represents the total reservoir outflow. The arc from node 1 to node 4 represents the local runoff downstream of the reservoir. The arc from node 4 to node 2 carries water from the reservoir/demand point network to the sink.

(4) To analyze multiple-period system operation, a layered network is developed. Each layer represents one month. To develop such a layered network, the single-period network representation is duplicated for each time period to be analyzed. Figure B-2 illustrates this. A single source node and a single sink node are included. For clarity, these have been omitted from the figure. The duplicate networks are connected by arcs that represent reservoir storage. For example, in Figure B-2, the arc connecting node 3 in period 1 to node 3 in period 2 represents the storage. The flow along this arc is the end-of-period 1 storage. This is equivalent to the beginning-of-period 2 storage. Likewise, the flow along the arc connecting node 3 in period 2 to node 3 in period 3 represents the end-of-period 2 storage. This also is the beginning-of-period 3 storage.

b. Formulate the allocation problem as a minimum-cost network-flow problem.

The goals of and constraints on water allocation within the reservoir system can be represented in terms of flows along the arcs of the network. If a unit cost is assigned for flow along each arc, the objective function for the network is the total cost for flow in all arcs. The ideal operation is that which minimizes this objective function while satisfying any upper and lower bounds on the flow along each arc. The solution also must maintain continuity at all nodes.

(1) Minimum-cost objective function.

(a) A network solver finds the optimal flows for the entire network simultaneously, based on the unit cost associated with flow along each arc. The functions that specify these costs are defined by the analyst.

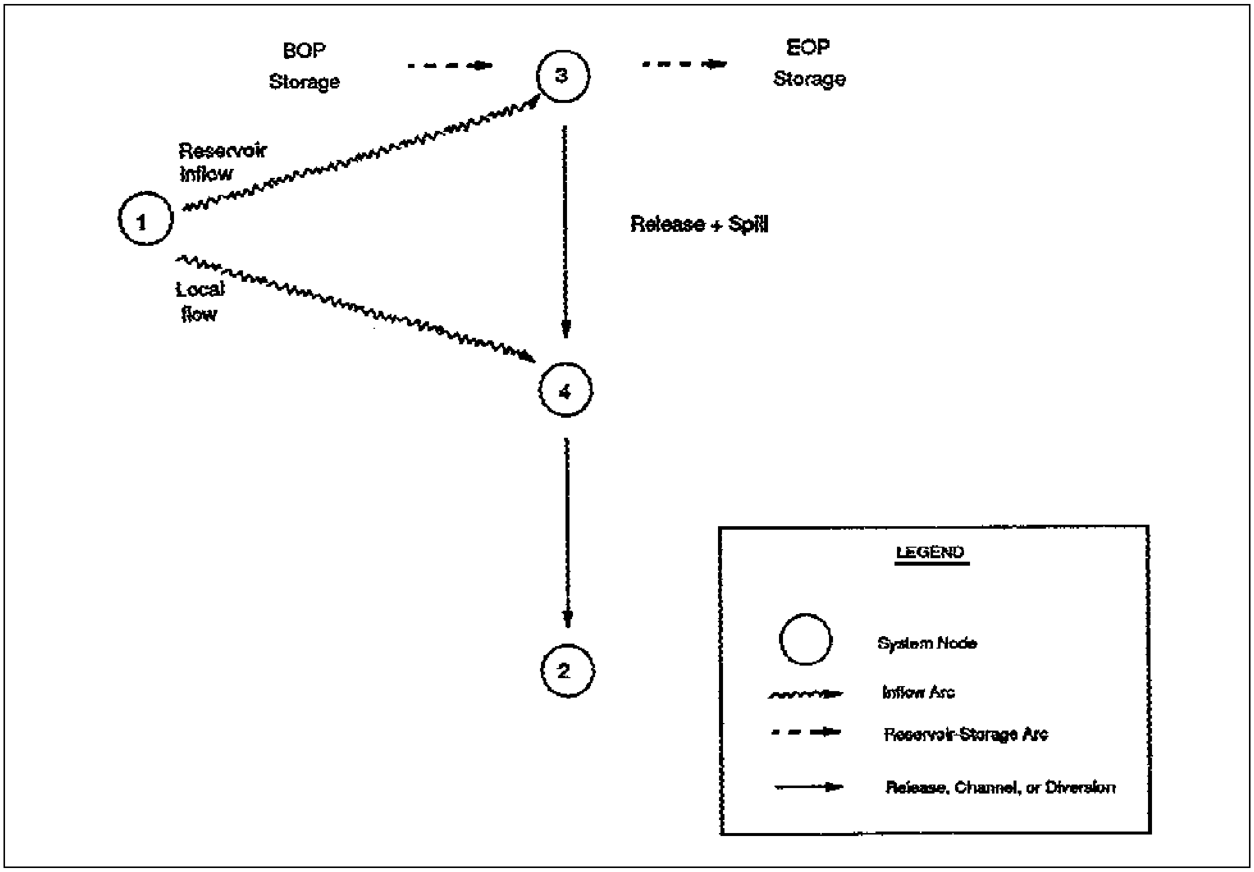

Figure B-1. Simplified single-period network

(b) The simplest cost function is a linear function, such as the one shown in Figure B-3. This function represents the cost for flow along one arc of a network. The cost increases steadily as the flow increases in the arc. The unit cost is the slope of the function. Here, it is positive, but it may be positive or negative. The total cost for flow along the arc represented is the product of flow and the unit cost.

(c) The simplest linear function may be too simple to represent adequately many of the goals of reservoir operation. Instead, nonlinear functions, such as those shown in Figure B-4, may be required.

(2) Piecewise-linear approximation.

(a) If the cost functions are convex, as are those in Figure B-4, they can be approximated in a piecewise-linear fashion for the proposed network model. Figure B-5 illustrates piecewise approximation of a complex cost function. Linear segments are selected to represent the pertinent characteristics of the function. The analyst controls the accuracy of the approximation. More linear segments yield a more accurate representation. However, the time required for solution of the resulting network-flow programming problem depends on the number of arcs included in the network. Thus, as the approximation improves, the time for solution increases. Jensen and Barnes discuss this approximation in detail (1980, pp. 355-357). HEC-PRM has an interactive utility program (**PENF**) to develop linear approximations of penalty functions defined in an HEC-DSS file. The user can select the number of segments to use and the program will provide the "best" segments and an error value for the approximation.

(b) With a piecewise linear approximation, the physical link for which the function applies is represented in the network by a set of parallel arcs. One arc is included for each linear segment of the piecewise approximation. For example, suppose the cost function in Figure B-5 represents the cost of release from the reservoir represented by node 3 in

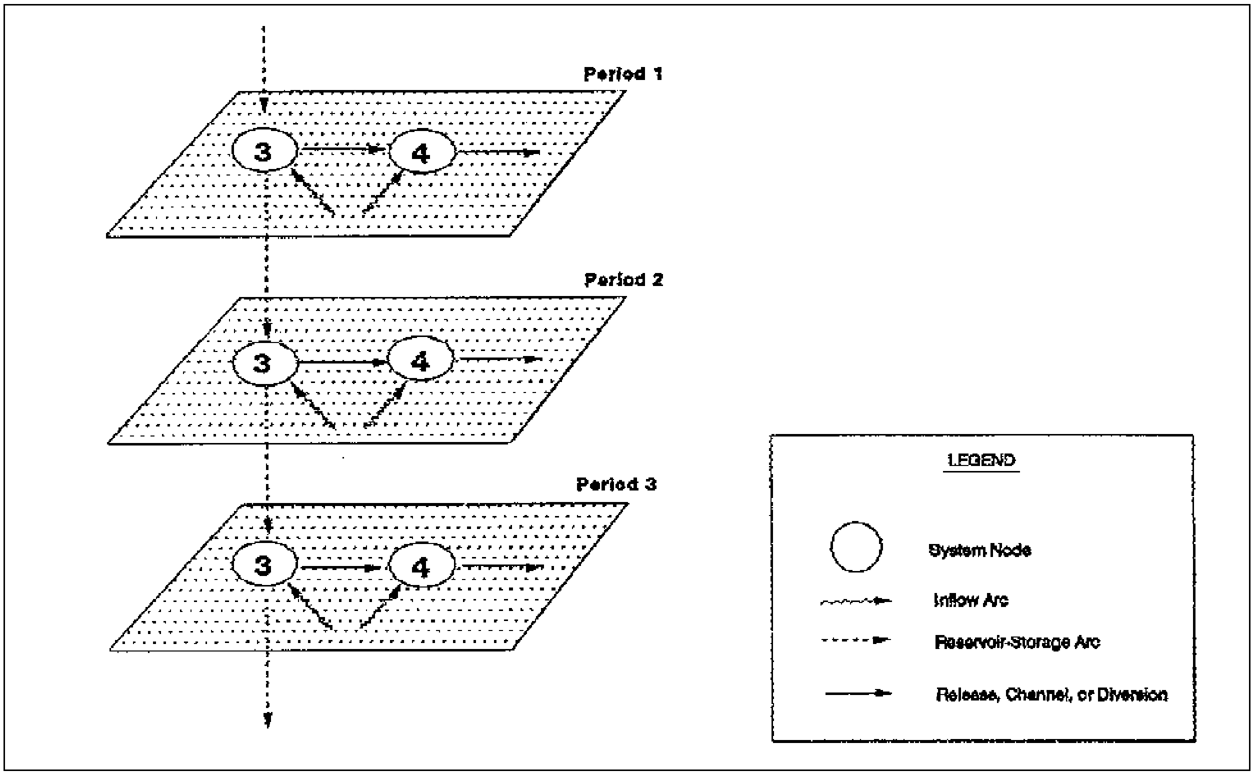

Figure B-2. Multiple period network

Figure B-1. In the proposed network model, four parallel arcs will connect node 3 to node 4. Characteristics of the arcs are shown in Table B-1.

(c) Arc 1 has the least marginal cost. Therefore, as flow is increased from node 3 to node 4, flow will pass first through arc 1. When the capacity of this arc is reached, flow begins to pass through arc 2. Arc 3 will have non-zero flow if, and only if, arc 2 is at its upper bound. Finally, arc 4 will have non-zero flow only when arcs 1, 2, and 3 are flowing full. Because the objective is to minimize cost, if two or more arcs are parallel, the one with the lowest unit cost is used first.

c. Develop objective function representing desirable operation.

(1) Penalty functions.

(a) The value functions are cost-based penalty functions that show the loss in economic value as the flow or storage in each model link deviates from the optimum. For the Columbia River System Model, cost-based penalty functions were developed for hydropower, flood damage, navigation, water supply/irrigation, recreation, and anadromous fish. *Economic Value Functions for Columbia River System Analysis Model, Phase 2,* describes the development of the individual penalty functions for the study (IWR 1993).

(b) Not all system operation goals can be represented adequately with economic costs. Some of the goals are socially, environmentally, or politically motivated, and the cost of failing to meet these goals is not amenable to economic analysis. If the achievement of these goals can be defined by limits on flow or storage, they can be expressed as constraints in the model. However, constraints reduce system flexibility to meet other system goals. Alternatively, the goals can be defined as penalty functions; for example, no penalty for the desired range of flow or storage and an increasing cost for flow or storage beyond the desired range. Paragraph B-5 describes penalty functions for typical reservoir purposes, including non-cost-based penalties.

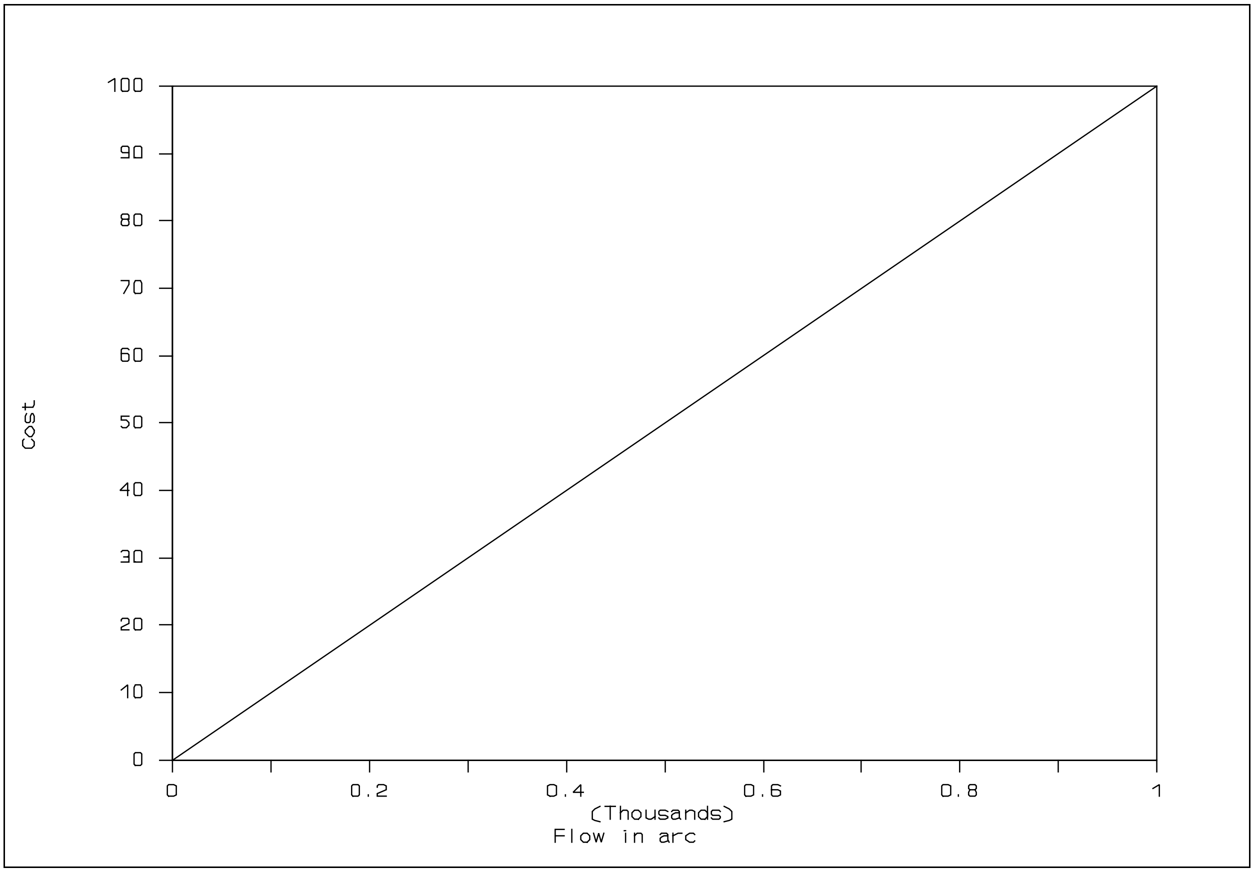

Figure B-3. Simple linear cost function

(c) The network flow model can only accept one penalty function per month per arc. The cost-based penalties are summed into a composite function for each location and month. The non-cost-based penalty functions must either be added to the cost-based functions to arrive at a single function for each arc, or assigned to a separate arc. Paragraph B-5*g* describes combining penalty functions. While HEC-PRM works with combined penalty functions, the program package includes post-processing to compute the penalties for each purpose (penalty function) and a utility program (**PRMPP**) to provide time-series displays and statistics for each purpose and for system totals.

(2) Flow penalty functions.

(a) All operation goals related to reservoir-release, channel-flow, or diversion-flow are expressed with flow penalty functions. These functions may represent operation goals for navigation, water supply, flood control, or environmental protection.

(b) Figure B-6 is an example of a flow penalty function. This function represents the relative penalty for diverting flow when the minimum desired diversion is 100 cfs. Less diversion is undesirable and the cost increases. More diversion is acceptable, but that water does not reduce the penalty.

(c) The penalty function of Figure B-6 is represented in the network by two parallel arcs. The characteristics of these arcs are shown in Table B-2.

(d) The first arc represents flow up to the desired rate. As the flow increases from 0 cfs to 100 cfs, the total penalty decreases. At 100 cfs, the unit penalty is 0.00. As the flow increases beyond 100 cfs, the unit penalty remains 0.00. Similar penalty functions can be developed for reservoir release and channel flow.

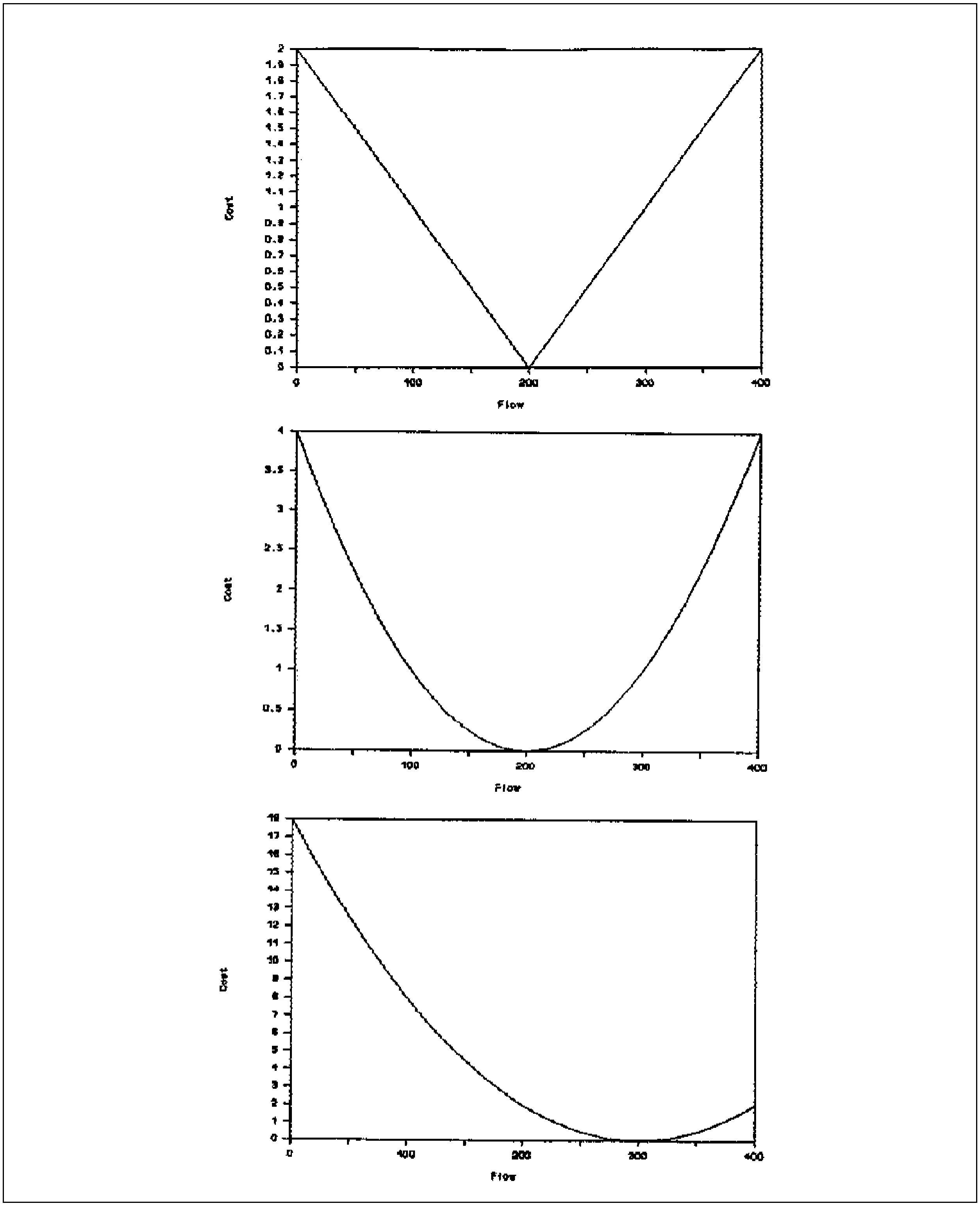

Figure B-4. Nonlinear penalty functions

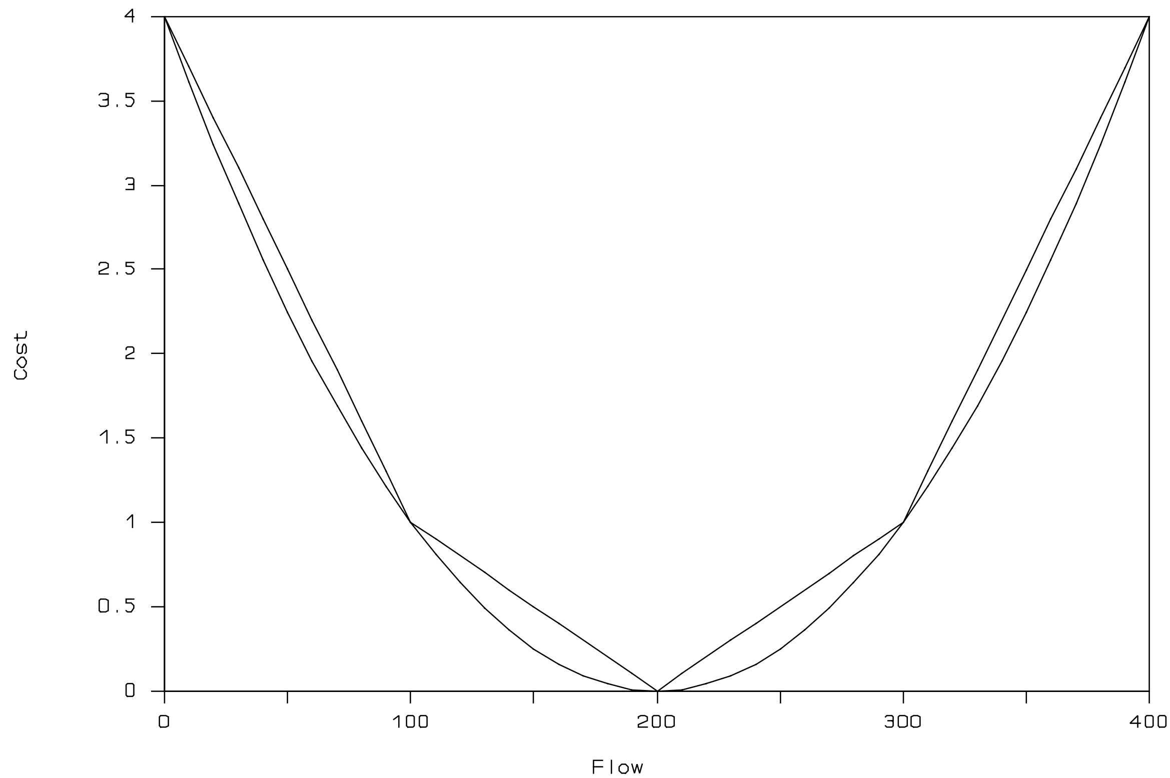

Figure B-5. Piecewise linear approximation of nonlinear penalty function

Table B-1
Example Network Model Arc Characteristics

Arc Number	Lower Bound	Upper Bound	Unit Cost
(1)	(2)	(3)	(4)
1	0	100	(1-4)/100=-0.03
2	0	200-100=100	(0-1)/100=-0.01
3	0	300-200=100	(1-0)/100= 0.01
4	0	400-300=100	(4-1)/100= 0.03

(3) Storage penalty functions.

(a) All reservoir operation goals uniquely related to storage are expressed through penalty functions for arcs that represent reservoir storage. Typical reservoir functions may include reservoir recreation, water supply, or flood control.

(b) Figure B-7 is an example of a reservoir storage penalty function. For this example, the top of the permanent pool is 200 kaf, the top of the conservation pool is 800 kaf, and the top of the flood-control pool is 1,000 kaf. The function represents penalty for storage when the reservoir operation goal is to keep the inactive and conservation pools full and the flood control pool empty.

(c) The function of Figure B-7 is represented in the network by three parallel arcs. The flow along one arc represents storage in the permanent pool. Increasing the flow along this arc reduces the penalty rapidly. Flow along the second arc represents storage in the conservation pool. Increasing flow along this arc also decreases the penalty, but not as rapidly as does flow along the inactive-pool arc. The third arc represents storage in the flood-control pool. Increasing flow along the flood-control pool arc increases the penalty. The solver will allocate flow to the arcs to minimize the total system penalty: first to the inactive-pool arc, then to the conservation-pool arc, and finally to the flood-control pool arc.

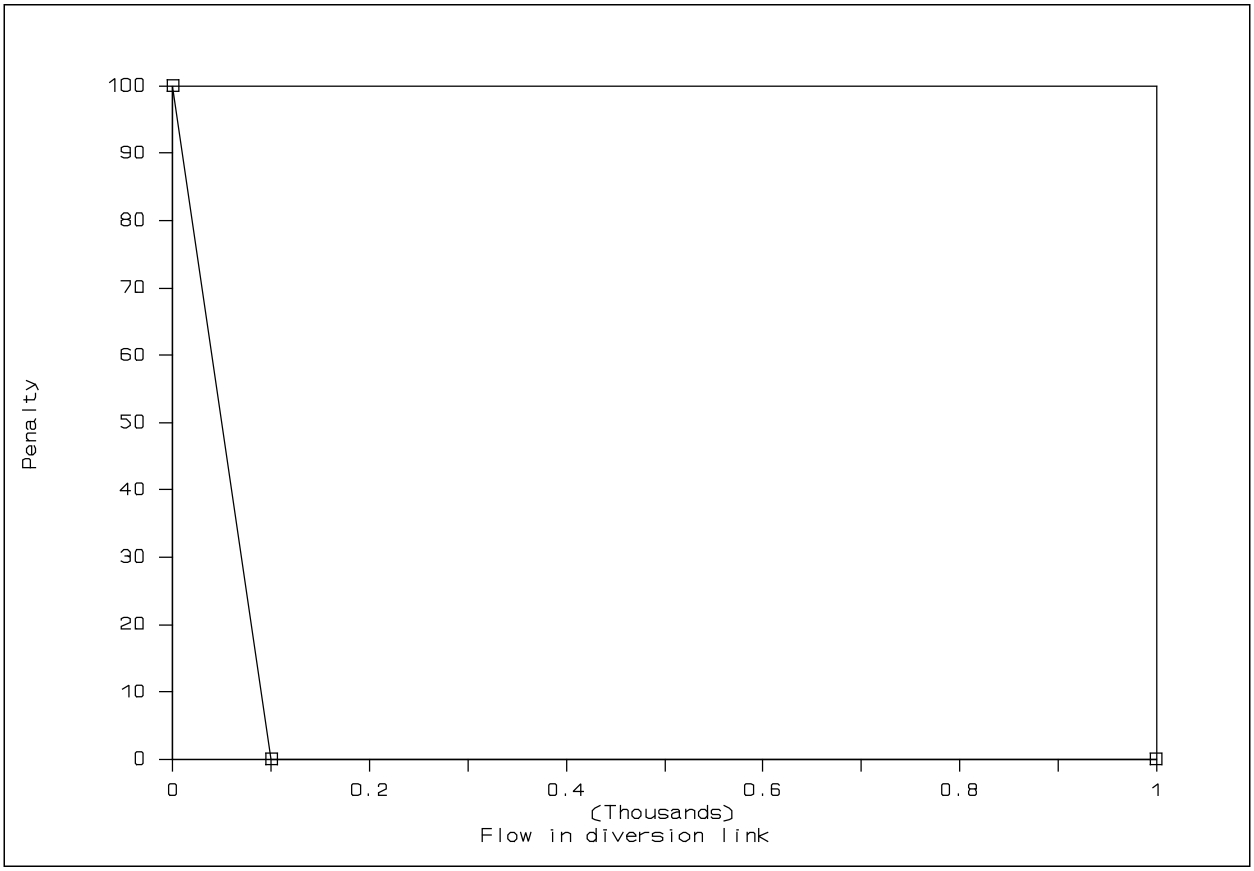

Figure B-6. Typical flow penalty function

Table B-2
Penalty Function Arc Parameters

Arc Number	Lower Bound	Upper Bound	Unit Cost
(1)	(2)	(3)	(4)
1	0	100	(0-100)/100=-1.00
2	0	1,000-100=900	0.00

(4) Storage and flow penalty functions.

(a) Certain system operation goals depend on both storage and flow. The most significant is hydroelectric energy generated at a reservoir. This is a function of the product of release and head on the turbine. Head is the difference in reservoir-surface elevation and downstream water-surface elevation. Reservoir-surface elevation is a function of reservoir storage, and downstream water-surface elevation is a function of release. Thus, the energy generated is a complex function of storage and flow.

(b) Figure B-8 illustrates the hydropower energy penalty function. Here, penalty is measured in terms of reduction in value of the energy produced, when compared to the firm energy target. Additional energy generated has a value, but that value is less than firm energy. Thus the slope is less.

d. Solve the network problem with a commercial solver.

(1) Mathematical statement of problem. The optimization problem represented by the network with costs associated with flow can be written as follows (Jensen and Barnes 1980):

Minimize

$$\sum_{k}^{m} h_k \, f_k \qquad \text{(B-1)}$$

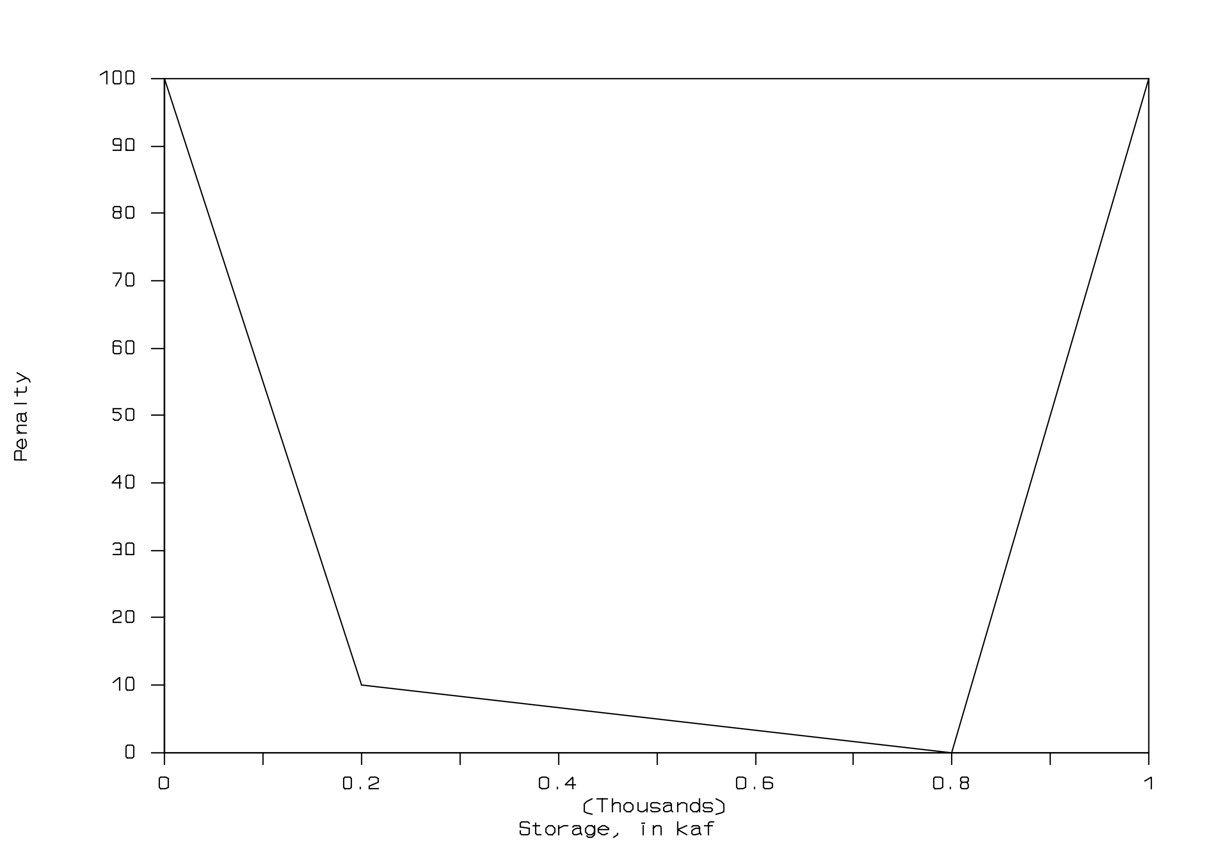

Figure B-7. Typical storage penalty function

subject to

$$\sum_{k \varepsilon M_O} f_k - \sum_{k \varepsilon M_T} a_k f_k = 0 \quad \text{(for all nodes)} \tag{B-2}$$

$$l_k \leq f_k \leq u_k \quad \text{(for all arcs)} \tag{B-3}$$

in which

k = an index of the arcs

m = total number of network arcs

h_k = unit cost for flow along arc k

f_k = flow along arc k

M_O = the set of all arcs originating at a node

M_T = the set of all arcs terminating at a node

a_k = multiplier for arc k

l_k = lower bound on flow along arc k

u_k = upper bound on flow along arc k

Equations B-1, B-2, and B-3 represent a special class of linear-programming (LP) problem: the *generalized minimum-cost network-flow problem*. Solution of the problem will yield an optimal allocation of flow within the system.

(2) Network solvers.

(a) Jensen and Barnes (1980) describe a variety of solutions to the generalized minimum-cost and other network-flow programming problems. One solution is the flow-augmentation algorithm developed by Jensen, Bhaumik and Driscoll (1974). This algorithm determines the minimum-penalty flow in a

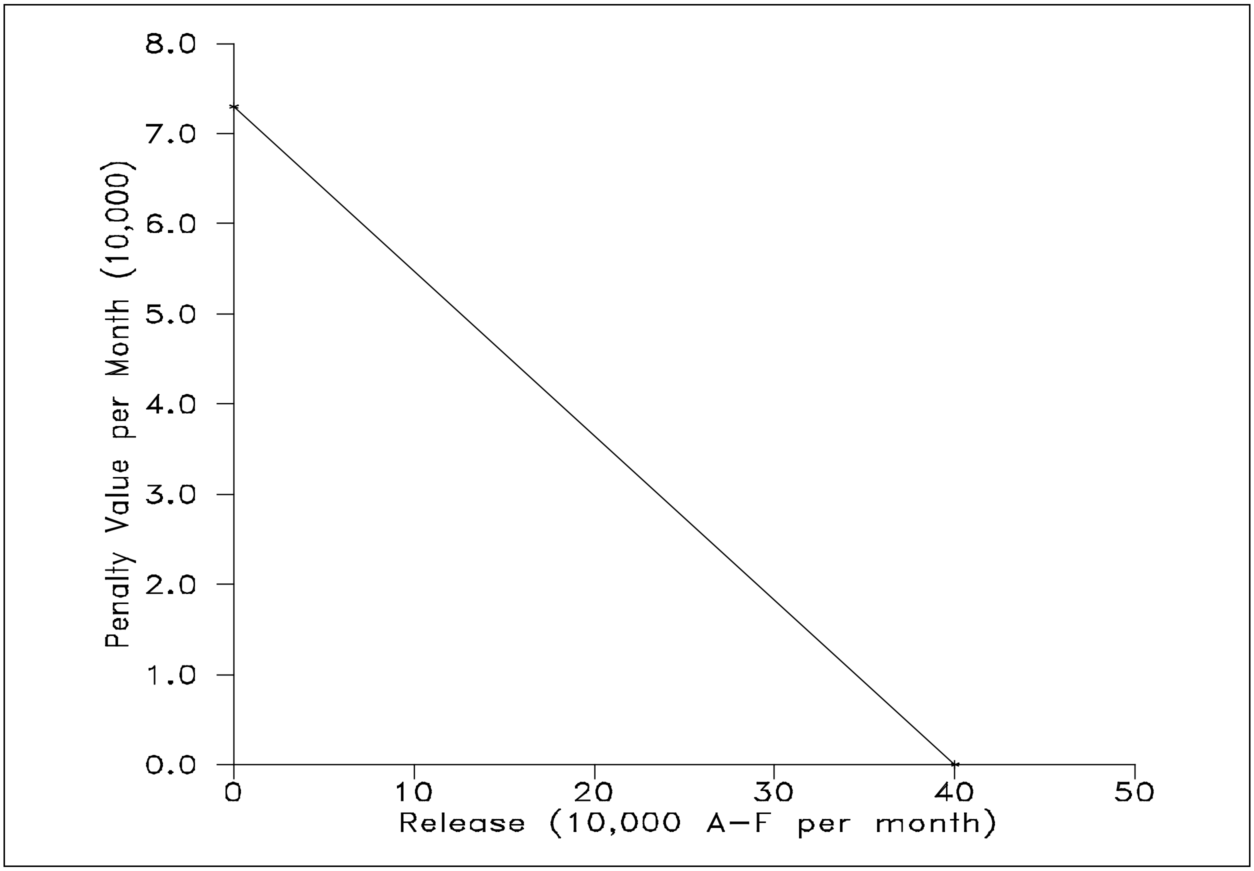

Figure B-8. Typical hydropower energy penalty function

generalized network by iteratively performing two computations. In the first computation, at the first iteration, the algorithm solves a shortest-path problem. That is, it determines a set of arcs that provide the minimum-penalty path from the source node to the sink node. In each successive iteration, the shortest-path computation deletes an arc with flow at upper bound from the path. It then adds the most promising available arc to create a new path. The second computation determines the maximum flow that can be directed from source to sink through the current shortest path. It increases flows in the arcs to achieve the maximum possible flow at the sink. If this flow equals an analyst-specified flow requirement at the sink, the algorithm terminates. Otherwise, the algorithm continues with the first computation. FORTRAN routines implementing this algorithm were published by Jensen and Bhaumik and used by Martin (1982). These routines were enhanced by Jensen, under contract with HEC (Jensen 1991a & b). The improved solver is presently used in the HEC-PRM.

(b) If a_k = 1.00 for all k in Equation B-2, the resulting problem is a *pure network-flow programming problem*. For this class of problem, faster solution algorithms are available. The well-known out-of-kilter algorithm (OKA) (Fulkerson 1961) solves this pure network problem. A FORTRAN routine implementing the OKA has been available as shareware since 1967 (SHARE). Barr, Glover, and Klingman (1974) presented an improved formulation of the OKA and developed a FORTRAN code to implement their algorithm. They present results showing that the reformulated algorithm is faster than the share routine by a factor of 4 to 15 on large problems. This code, designated SUPERK, is published by the Texas Department of Water Resources (1975) and used by the California Department of Water Resources (Chung, Archer, and DeVries 1989). FORTRAN code for SUPERK is available at HEC.

(c) Karney and Klingman (1976) present a special-purpose in-core, out-of-core code for solving capacitated transhipment and transportation network problems. They report that this code has solved problems with 50,000 nodes and 62 million arcs on a UNIVAC 1108 for the U.S. Treasury Department. They also report solution of networks with 625,000 arcs on machines with less than 30,000 words of central memory. This code, designated I/O PNET-I, is available commercially.

e. Post-process network results.

(1) The optimal allocation of water in the layered network is determined with a network solver. The solver finds the flow along each network arc that yields the total minimum-penalty circulation for the entire network, subject to the continuity and capacity constraints. These flows must be translated into reservoir releases, hydropower generation, storage volumes, diversion rates, and channel flows to be useful to the reservoir system operators.

(2) For convenience, the results after translation are stored with the HEC data storage system (HEC-DSS). Then the results can be displayed or processed further to provide information required for decision making. A utility program has been developed to facilitate the development of output tables. The program also provides graphical display of time-series data written to the DSS file.

B-4. Model-Building Software

To the extent possible, the software to implement the network model is general-purpose software. With this software, an analyst is able to define the layout of any existing or proposed reservoir system. Further, the analyst is able to describe the physical features of the system reservoirs and channels and the goals of and constraints on their operation. The operation goals are defined by penalty functions associated with flow, storage, or both. The reservoir system is represented as a network which includes the following model-building components:

- Inflow link
- Initial-storage link
- Diversion link
- Final-storage link
- Channel-flow link
- Simple reservoir-release link
- Hydropower reservoir-release link
- Reservoir-storage link
- Nodes at which links are connected

By selecting the appropriate links and the manner in which they are interconnected, the analyst can describe any system. By describing the characteristics of the links and the penalties associated with flow along the links, the analyst can define operating constraints and goals.

a. Inflow link.

(1) An inflow link brings flow into the reservoir-system network. It originates at the source node and terminates at any other system node. In Figure B-1, the link from node 1 to node 3 is an inflow link. It originates at the source node, node 1, and carries flow into the system at node 3.

(2) The flow along the arc representing the inflow link is an input to the model. This known inflow may be an observed inflow from the historical record, or it may be an inflow from a sequence generated with a statistical model. To ensure that the link carries the specified flow, the arc upper and lower bounds are equal, and the unit penalty is zero.

b. Initial-storage link.

(1) An initial-storage link is a special case of an inflow link. It originates at the source node and terminates at a node that represents a reservoir in the first period of analysis only. It introduces to the network the volume of water initially stored in the reservoir. In Figure B-2, the storage link terminating at node 3 in period 1 is an initial-storage link; it represents the beginning-of-period 1 storage.

(2) As an initial-storage link carries a specified flow, no decision is represented by this link. To ensure that the link carries the specified flow, the arc upper and lower bounds are equal, and the unit penalty is zero.

c. Diversion link.

(1) A diversion link carries flow out of the system. It originates at any system node and terminates at the sink node. In Figure B-1, the arc from node 4 to node 2 is a diversion link. It originates in the system at the downstream control point, node 4. It carries flow out of the system to the sink, node 2.

(2) The flow along a diversion link is a decision variable, selected to minimize total system penalty. The diversion penalty function is specified by the analyst as a convex piecewise approximation of the true penalty associated with deviating from the diversion desired. This function may vary by month. The software will define appropriate arc bounds and unit costs to represent the function.

(3) The analyst may specify also inviolable minimum and/or maximum flow for a diversion link. If the analyst specifies both minimum and maximum, and if these values are the same, the diversion link is represented in the network by a single arc. The upper and lower bounds of the arc are equal. In that case, the only feasible solution is one in which flow equals the specified value, regardless of cost. Any penalty function defined by the analyst for the link is ignored in that case, as it has no impact on the solution.

(4) If the analyst specifies only a lower bound or only an upper bound, the software will impose the bound on the appropriate network arcs. If the penalty function is a simple function, like that of Figure B-3, the bound is applied to the single arc representing that function. For example, if the analyst specified a lower bound of 25 cfs and an upper bound of 800 cfs, the network arc will have l_k = 25 and u_k = 800 (see Equation B-3).

(5) For more complex penalty functions, the software must include an algorithm to determine the proper network arcs on which to impose the bound. For example, the penalty function of Figure B-6 is represented by two parallel arcs, with bounds and cost. If the analyst specifies an inviolable lower bound of 25 cfs and an upper bound of 800 cfs, the network arcs must be adjusted to have the parameters shown in Table B-3.

Table B-3
Diversion Link Arc Characteristics

Arc Number	Lower Bound	Upper Bound	Unit Cost
(1)	(2)	(3)	(4)
1	25	100	-1.00
2	0	800-100=700	0.00

For the first arc, the lower bound increases from 0 to 25. The upper bound remains 100. The unit cost does not change. For the second arc, the lower bound remains 0, and the upper bound now is 800-100 = 700. The unit cost does not change.

d. Final-storage link.

(1) A final-storage link is a special case of a diversion link. It carries flow out of the system, but only from a reservoir in the last period of analysis. The final storage link thus originates at any system reservoir and terminates at the sink node. In Figure B-2, the storage link originating at node 3 in period 3 is a final-storage link. The final-storage link is included in the system model to permit assignment of a future value for water in system reservoirs. Otherwise, the network solver is indifferent regarding final storage. The solver may choose any storage state, including empty or full, without regard for future use.

(2) Just as with the diversion link, the flow along a final-storage link is a decision variable, selected to minimize total system penalty. The penalty function is specified by the analyst as a convex piecewise approximation of the true penalty associated with deviating from an ideal final storage. The software will define appropriate arc bounds and unit costs to represent this function.

(3) As with the diversion link, the analyst may specify also inviolable minimum and/or maximum storage for a final-storage link. The software will impose these constraints on the appropriate network arcs.

e. Channel-flow link.

(1) A channel-flow link originates at any non-reservoir node, terminates at any other network node, and represents the flow in a channel reach. The flow along the link is a decision variable, selected to minimize total system penalty.

(2) As with the diversion link, the analyst may specify inviolable minimum and/or maximum flow for a channel-flow link. The software will impose these constraints on the appropriate network arcs.

(3) The analyst may specify also a multiplier for flow along a channel-flow link. The multiplier is a_k of Equation B-2 for all arcs representing the link. If the multiplier is greater than 1.00, it represents increase of flow in the channel. If the multiplier is less than 1.00, it represents loss of flow.

f. Simple reservoir-release link.

(1) The reservoir-release link originates only at a non-hydropower reservoir node, terminates at any other node, and represents the total outflow from a reservoir. This includes release and spill. The flow along a reservoir-outflow link is a decision variable, selected to minimize total system penalty. In Figure B-1, the link from node 3 to node 4 is a simple reservoir-release link. It originates at a node representing a reservoir and terminates, in this case, at a node representing a demand point.

(2) The analyst may specify inviolable minimum and/or maximum flow constraints. The analyst may specify also a multiplier for flow along a reservoir-release link. The software will apply the multiplier and impose the constraints on the appropriate network arcs.

g. Hydropower reservoir-release link.

(1) Link description.

(a) A hydropower reservoir-release link (hydro-release link) originates only at a hydropower reservoir node, terminates at any other node, and represents the total outflow from the reservoir. This includes release and spill.

(b) The flow along a hydro-release link is a decision variable, selected to minimize total system penalty. As hydroelectric energy is not a linear function of flow, however, determination of the release that minimizes total penalty requires consideration of storage.

(2) Hydropower computation from link flow.

(a) The nonlinear hydro-release problem is solved via iterative solution of linear approximations. Such successive linear programming techniques are described by Martin (1982), Grygier and Stedinger (1985), and Reznicek and Simonovic (1990). In summary, these techniques convert the energy penalty functions to release penalty functions by assuming a value of reservoir storage. Given the storage, head can be estimated. Given this head, the unit penalty for release is used, and the flow allocation problem is solved. Then the head assumption is checked, using the storage computed for the optimal allocation. If the assumption is not acceptable, the heads corresponding to the computed storages are used, and the process is repeated.

(b) The algorithm proposed by Grygier and Stedinger (1985) is employed in the proposed model. This algorithm solves the hydro-release problem as follows:

Step 1. Set ITER, an iteration counter, equal zero. Assign a value to ΔQ_{max}, the maximum allowable percentage release deviation.

Step 2. For each hydro-release link for each period, estimate the beginning-of-period (BOP) and end-of-period (EOP) storage for penalty calculation. Note that this may be a reservoir other than that upstream of the link.

Step 3. Determine the BOP and EOP head corresponding to the storage. Given the head, convert the energy penalty function to a flow penalty function. Assign the appropriate linear costs to the release and storage arcs. Add constraints to the release arcs so the release does not vary by more than ΔQ_{max} percent.

Step 4. Solve the resulting network flow programming problem.

Step 5. For each hydro-release link for each period, determine the average releases computed for the optimal network solution. Compare the computed values with the values used in step 2.

If all values are accurate within a user-specified tolerance, stop. Otherwise, go to step 6.

Step 6. If the objective function value is worse than the value found in the previous iteration, go to step 7. Otherwise, accept this solution. Determine from the optimal solution the BOP and EOP storage for each hydro-release link for each period. Set ITER = ITER + 1 and decrease ΔQ_{max} by one half. Repeat the computations, beginning with step 2.

Step 7. Decrease ΔQ_{max}. Repeat the computations, beginning with step 3, without updating the release estimates.

(3) Other release penalties. Due to the special nature of the hydro-release link, all other release-related penalties must be defined as a function of flow downstream. This is accomplished by defining a "dummy" node downstream of the hydropower reservoir. The hydro-release link connects the reservoir and this dummy node, and the hydropower penalty function is associated with this link. A channel-flow link connects the dummy node with the next downstream node. All penalty functions normally defined in terms of reservoir release are defined in terms of channel flow instead.

h. Reservoir-storage link.

(1) Link description.

(a) A reservoir-storage link originates at any reservoir node in a layered, multiple-period network. It represents the volume of water stored in the reservoir at the end of the period. The reservoir-storage link terminates at the node representing the same reservoir in the period following. The flow along a reservoir-storage link is a decision variable, selected to minimize total system penalty.

(b) For example, in Figure B-2, the arc from node 3 in period 1 to node 3 in period 2 is a reservoir-storage link. Flow along the arc leaving the period-1 layer represents reservoir storage at the end of period 1. Flow along the arc entering the period 2 layer represents reservoir storage at the beginning of period 2.

(2) Evaporation computation with link flow. To approximate reservoir evaporation, a fraction of flow entering the reservoir-storage link may be "lost." For the network model, the relationship of storage and evaporation is given by

$$S_t = S_{t-1} - EV_{t-1} \qquad \text{(B-4)}$$

in which

S_t = reservoir storage at beginning of period t

S_{t-1} = reservoir storage at end of period t-1

EV_{t-1} = volume of reservoir evaporation

The evaporation volume is related to reservoir surface area with the following equation:

$$EV_{t-1} = (ED_{t-1})\,(A_{t-1}) \qquad \text{(B-5)}$$

in which

ED_{t-1} = evaporation rate in period t-1

A_{t-1} = reservoir surface area in period t-1

The quantity ED_{t-1} is input to the model. It may be an historically observed evaporation rate, or it may be generated with a statistical model. The relationship of surface area and storage can be approximated with a linear function as

$$A_{t-1} = ß\, S_{t-1} \qquad \text{(B-6)}$$

in which ß = a linear coefficient. The value of ß is found from analysis of specified reservoir characteristics. Substituting Equations B-5 and B-6 into Equation B-4 and simplifying yields

$$S_t = (1 - ED_{t-1}\, ß)\,(S_{t-1}) \qquad \text{(B-7)}$$

The quantity $(1 - ED_{t-1}\, ß)$ is an arc multiplier. The flow out of the reservoir-storage arc S_t is the flow into the arc S_{t-1} multiplied by $(1 - ED_{t-1}\, ß)$. This multiplier is the arc multiplier a_k of Equation B-2. If the magnitude of $(1 - ED_{t-1}\, ß)$ is approximately 1.00 for all periods of analysis, $S_t = S_{t-1}$. That is, reservoir storage at beginning of period t = reservoir storage at end of period t-1. In that case, the network-flow programming is no longer a generalized network problem. Instead, it is a pure network problem. Faster solvers may be used.

i. Nodes.

(1) Nodes are included in the model to permit joining the appropriate links. Two or more of the links described may join at a node. The nodes represent system reservoirs, demand points, channel junctions, or diversion points. These may be existing facilities or proposed facilities. Additional nodes may be included in the network for convenience of description.

(2) In addition to the analyst-defined nodes, the software will incorporate in the network a source node and a sink node to satisfy the mathematical requirements for defining a network. All water entering the system flows from the source node. All water leaving the system flows to the sink node. These hypothetical nodes have unlimited capacity.

B-5. Typical Penalty Functions

The goals of reservoir system operation are identified by the analyst via penalty functions. The functions define, as a function of flow, storage, or both, the economic, social, and environmental cost for deviating from ideal operation for each of the system operation purposes. These purposes include flood control, navigation, lake and stream recreation, water supply, environmental protection, and hydropower.

a. Flood-control penalty function. A flood-control penalty function defines the cost of deviating from ideal flood-damage-reduction operation. This function typically will relate penalty to channel-link flow or reservoir release link flow. Figure B-9 is a typical flood-control penalty function. In this example, no penalty is incurred for flows less than 600 cfs, the channel capacity. Between 600 cfs and 1,100 cfs, the penalty is slight, increasing to 100 units. The penalty is much greater for flows exceeding 1,100 cfs. This represents significant damage incurred as the flow moves out of the 10- to 25-year floodplain and into surrounding property.

b. Navigation penalty function. A navigation penalty function defines the cost of deviating from flows desired for vessel traffic in a system channel. Figure B-10 is a typical navigation penalty function. In this example, the penalty is great for flows less than 400 cfs; this represents the minimum desired flow for towing barges in the channel. Between 400 and 600 cfs, the penalty is zero, as this is the desired flow for navigation. Between 600 and 1,100 cfs, the penalty increases slightly, representing the increased effort required for navigation. Finally, the penalty increases rapidly if the flow exceeds 1,100 cfs. This is the upper limit on desired flow for navigation.

c. Recreation penalty functions.

(1) A cost-based recreation penalty function may represent the relationship of recreation to reservoir storage or channel flow. Figure B-11 is an example of a typical lake recreation function. In this example, the desired range of active storage for recreation is 40 to 80 kaf. If the reservoir storage is less than 40 kaf, the boat ramps are inaccessible, and recreation is hazardous. If the reservoir storage is more than 80 kaf, the reservoir is in flood operation, and recreation is hazardous. Consequently, the function is shaped as shown.

(2) Figure B-12 is a typical river recreation penalty function. In this example, the desired range of flow for boating, swimming, and fishing is 400 to 500 cfs. As the flow rate drops below 400 cfs, boating and swimming become dangerous due to shallow depths and there is less area for fish. As the flow rate exceeds 500 cfs, recreation becomes hazardous.

d. Water-supply penalty function. A water-supply cost penalty function describes desired operation for supply of water for municipal and industrial use or for irrigation. A water-supply penalty function may relate to channel-link flow, simple reservoir-release flow, or diversion flow. Figure B-13 is a typical water-supply penalty function. In this function, the desired flow for water supply is 100 cfs. If the flow is less, demands are not met, so the penalty cost is great. If the flow exceeds the desired rate, there is no penalty cost to water supply.

e. Hydropower penalty function. A hydropower penalty function is assigned to a hydro-release link only and defines the cost of deviation from desired system operation for energy production. For the proposed model, Figure B-14 illustrates the acceptable form of the function. This function defines separate penalty curves for specified heads (storages). For each storage, the penalty function depends solely on the release. If the head is less

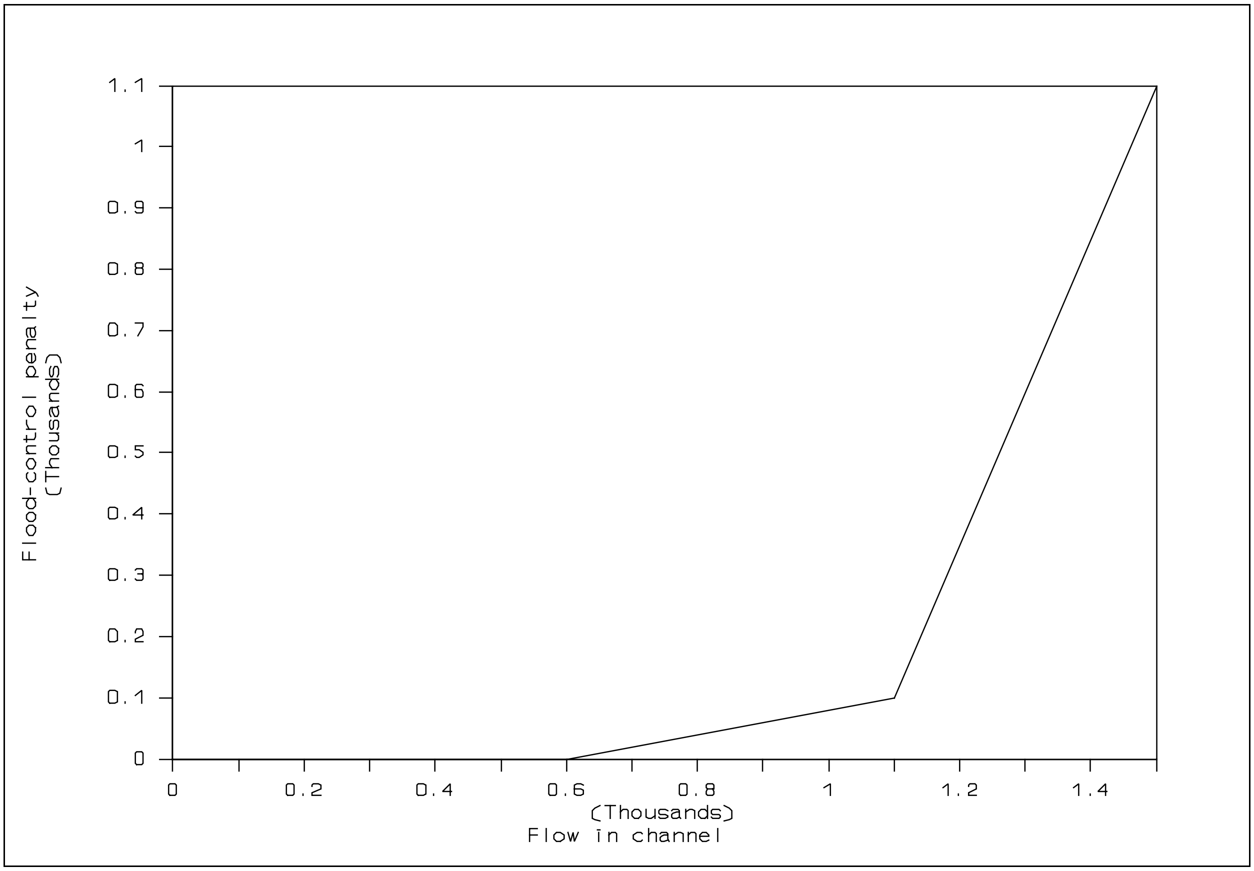

Figure B-9. Typical flood-control penalty function

than the optimal head for the generator, the penalty cost is positive. Likewise, if the release is less than optimal for a specified head, the penalty cost is positive.

f. Environmental penalty function. An environmental penalty function represents the desired operation for environmental protection. The non-cost based function may define a penalty for flow, or for storage, or for both. An example is illustrated by Figure B-15. In this case, an average flow of 25,000 to 30,000 cfs is required during one month to preserve a valuable wildlife habitat (1,500 to 1,800 - 1,000 A-F/month). If the flow is less or more, the habitat is destroyed. In that case, only the desired flow range is assigned zero penalty. For all other flows, the penalty is positive. This approach could also be used for other non-cost based goals.

g. Combined penalty functions.

(1) If two or more penalty functions apply to a single stream reach or to a single reservoir, the functions are combined to yield a single penalty function. The combined penalty function then is used in the optimization. For example, a flow link may have a penalty for flood control, water supply, navigation, energy, and recreation. To combine the cost-based functions, the various penalties for a given flow are added. The resulting function is then edited or smoothed to yield a convex function. This convex function then is represented in a piecewise linear fashion for the network. Figure B-16 illustrates this. A utility program has been developed to compute a piecewise-linear convex function for a computed combined penalty function. The program allows the user to select the number of linear elements to

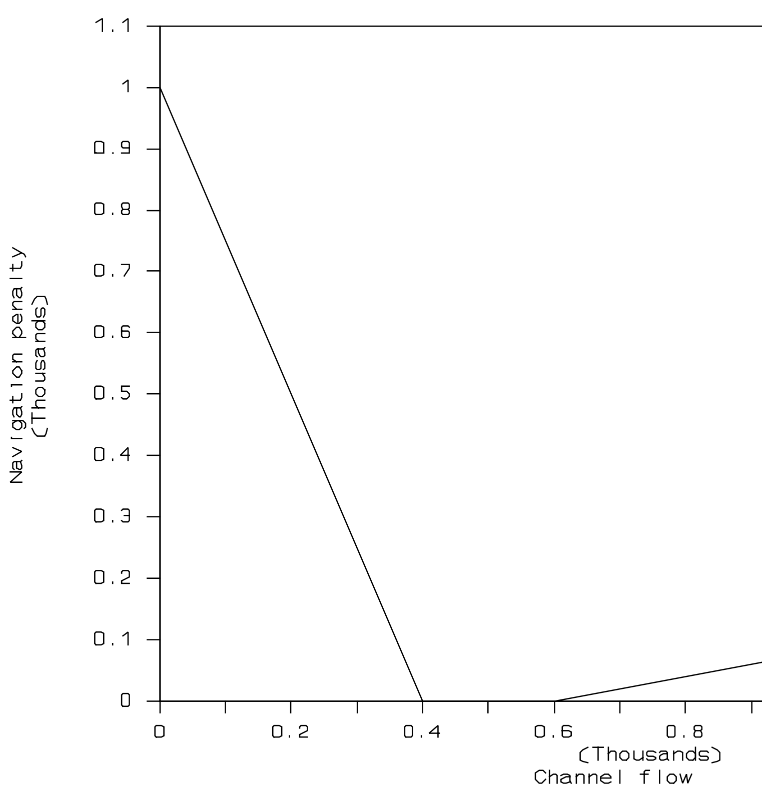

Figure B-10. Typical navigation penalty function

represent the function and to edit the function values, while ensuring that the function remains convex.

(2) Non-cost based penalty functions must be defined as a cost. They can be expressed as steep, V-shaped penalty functions with no penalty for values within the desired range, and increasing penalty cost for values too high or low (as shown in Figure B-15). These functions are not as restrictive as constraints because they allow deviations; however, the deviations incur very high penalties. These penalty functions are in commensurate units, but those units are not necessarily dollars. The penalty functions represent instead the relative economic, social, environmental, and political penalties associated with failure to meet operation goals. Thus, even if failure to meet, for example, an environmental operation goal has no measurable economic cost, the penalty may be great. Figure B-17 illustrates the addition of an environmental penalty with cost-based penalties. If non-cost based functions are used, the aggregate optimum system penalty cannot be interpreted in economic terms, and the cost-based and non-cost based penalties would need to be reported separately.

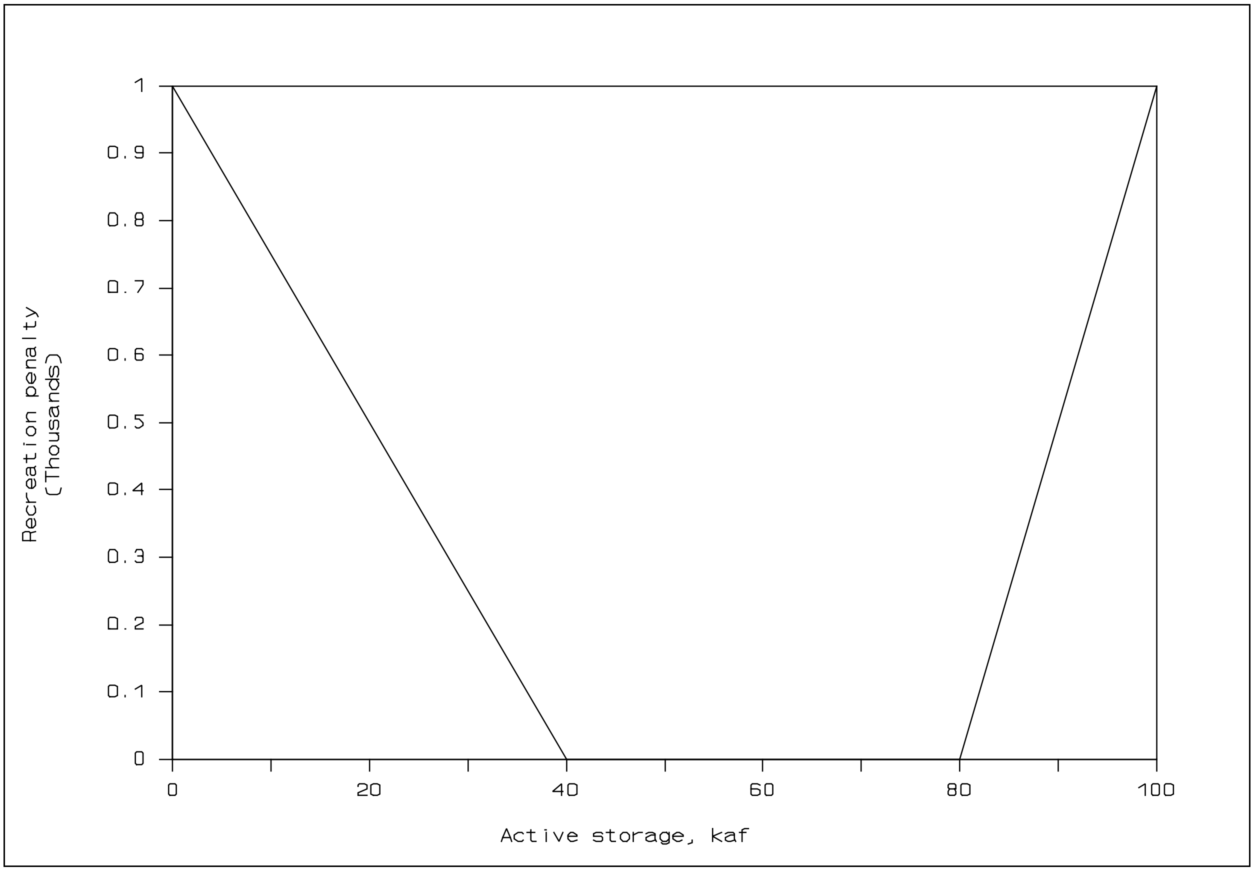

Figure B-11. Typical lake recreation penalty function

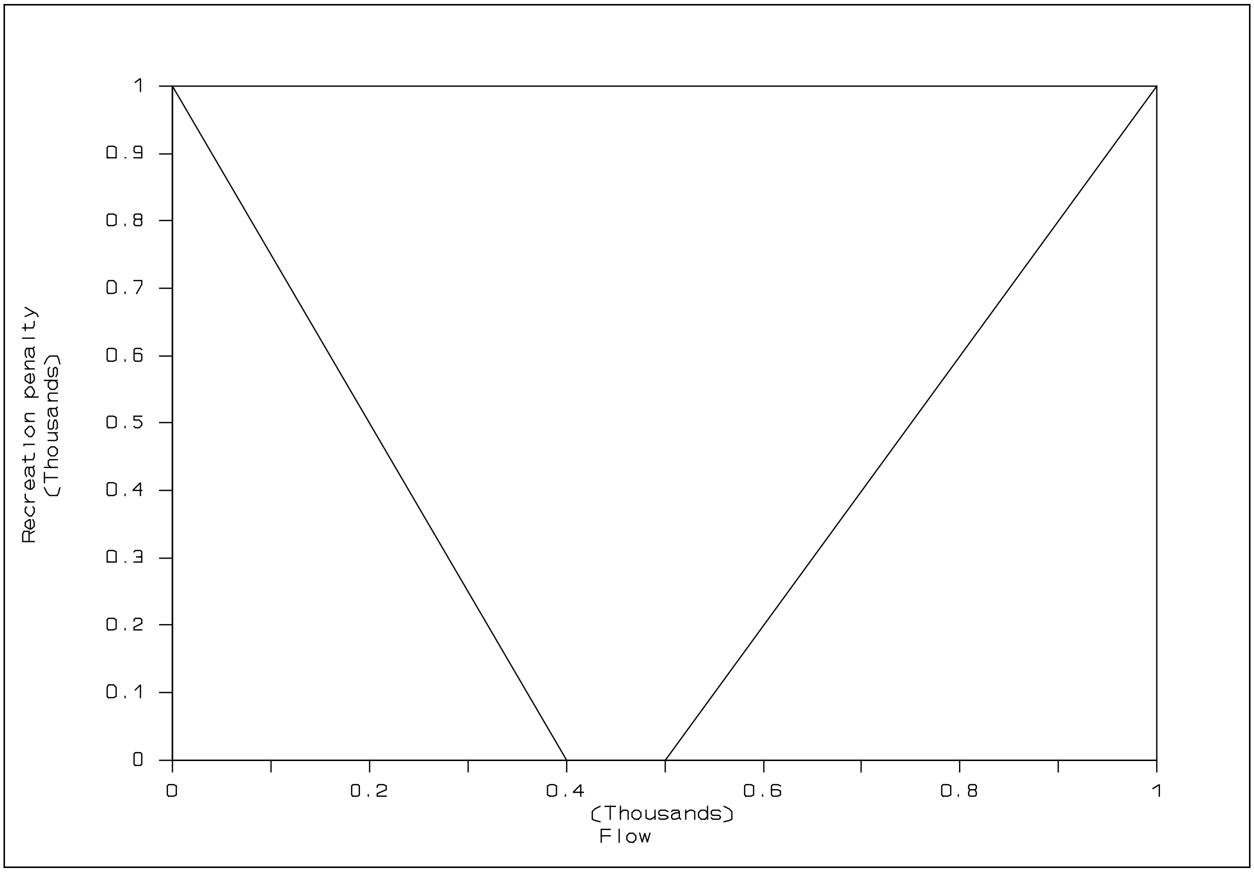

Figure B-12. Typical river recreation penalty function

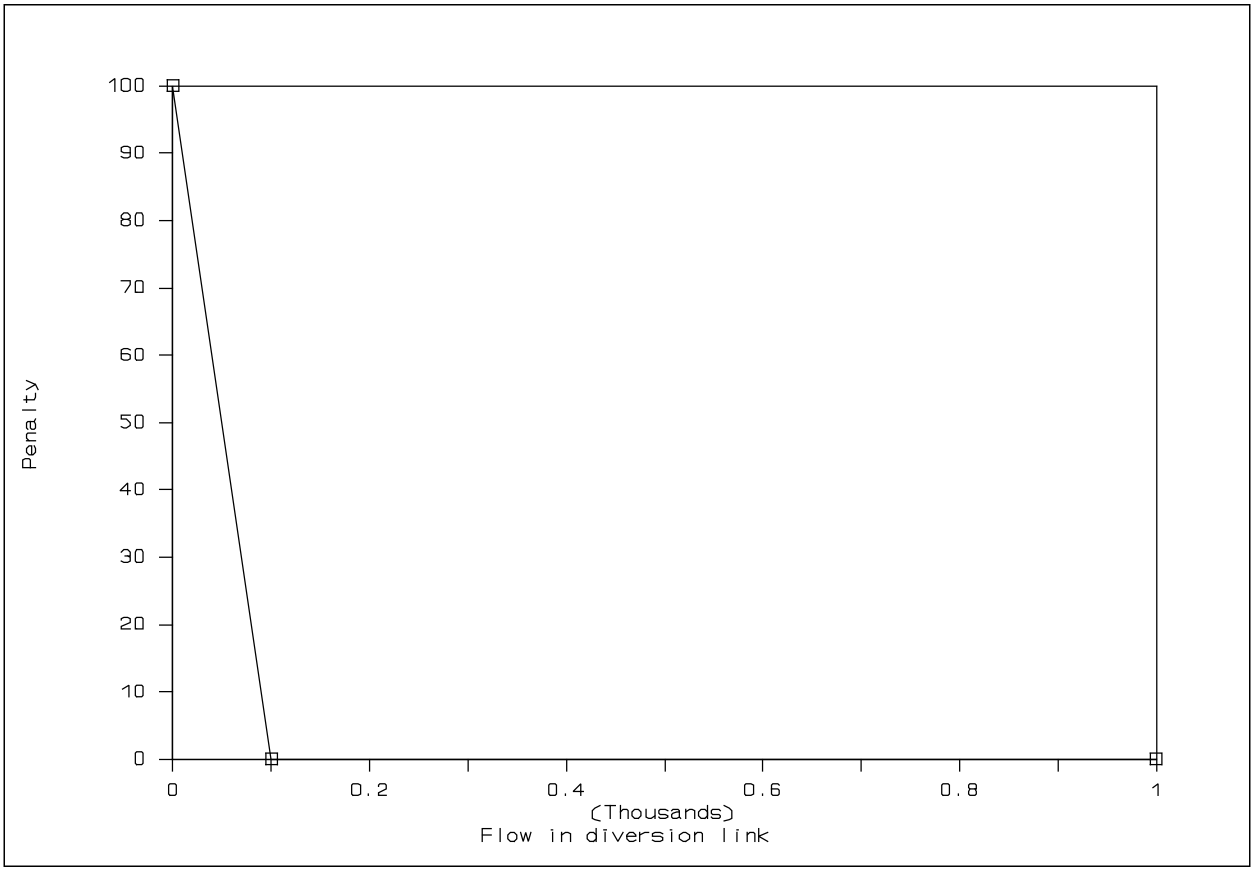

Figure B-13. Typical water-supply penalty function

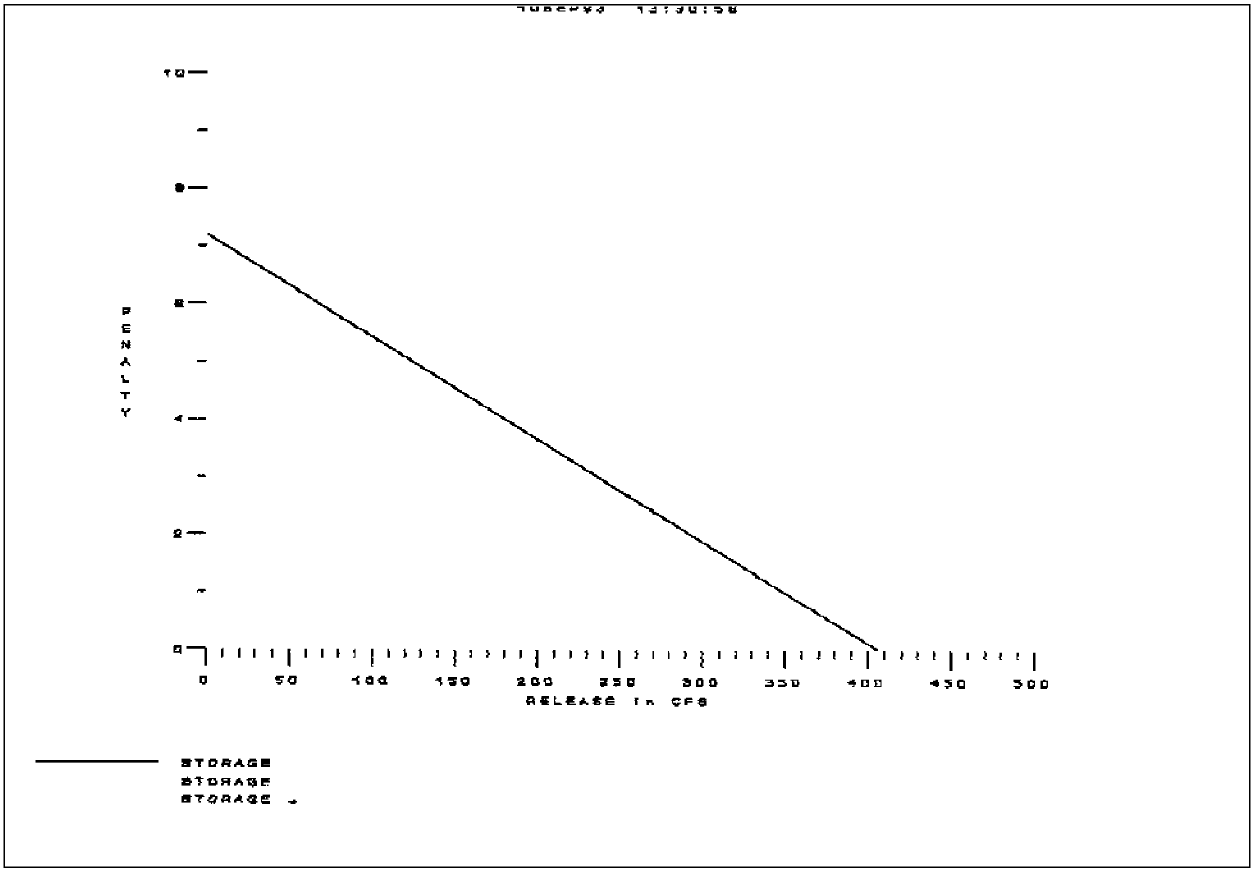

Figure B-14. Typical hydropower capacity penalty function

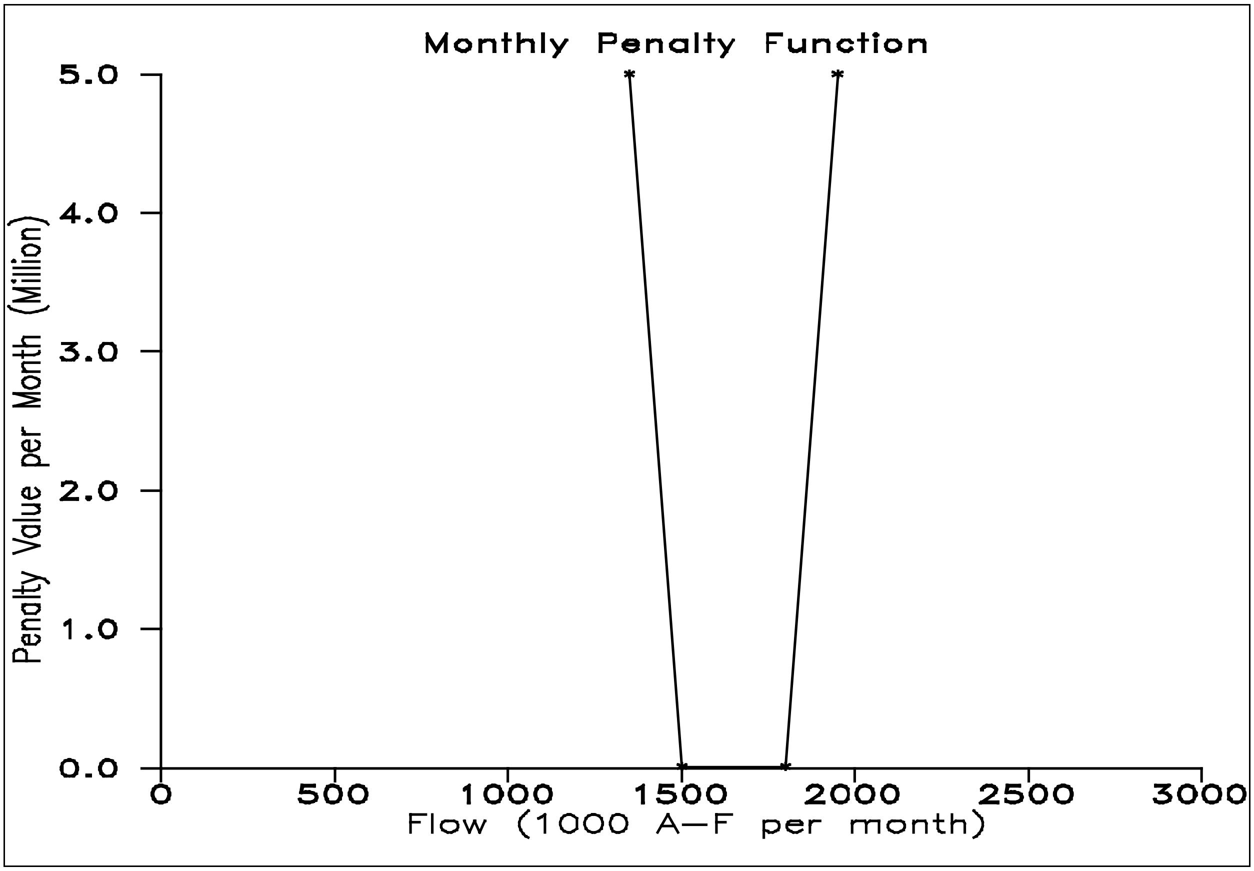

Figure B-15. Example environmental penalty function

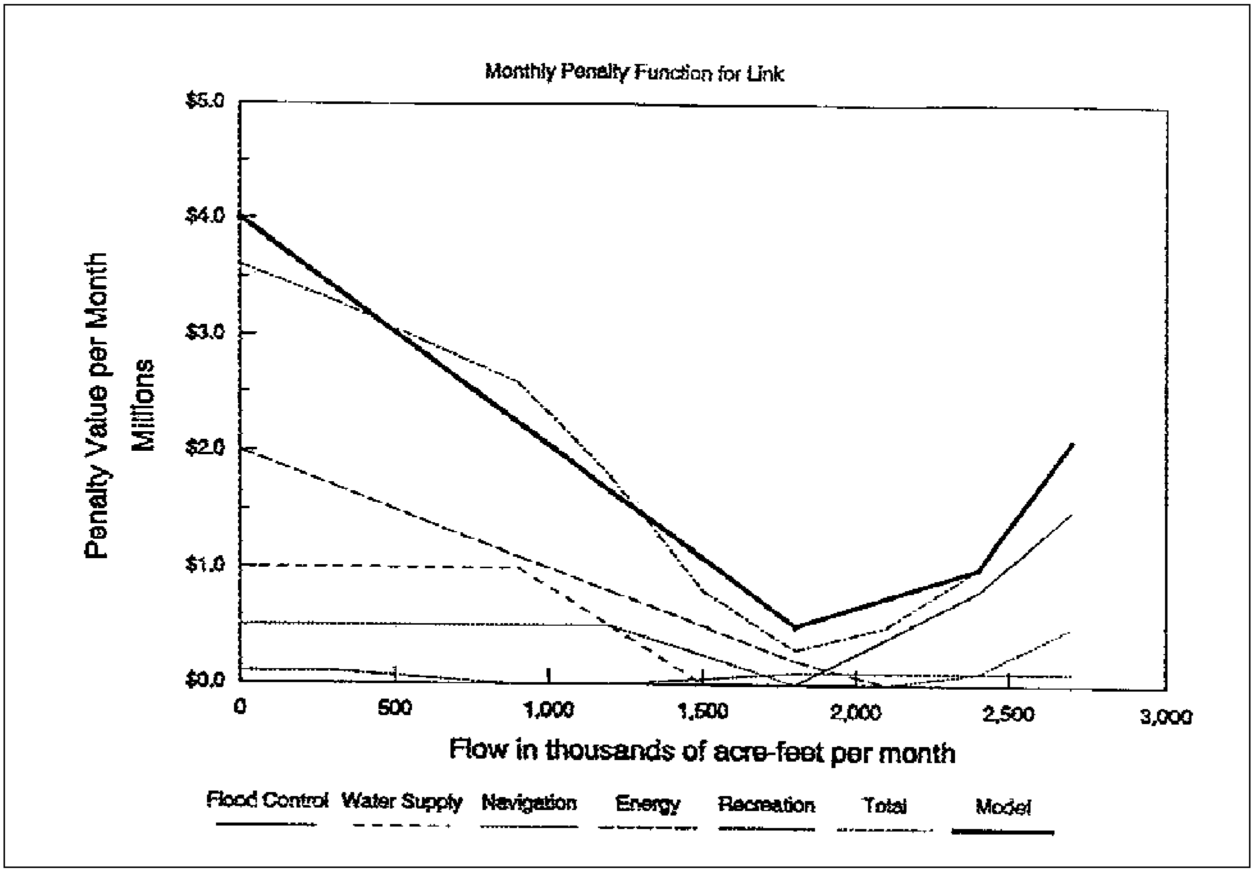

Figure B-16. Cost-based penalty functions combined

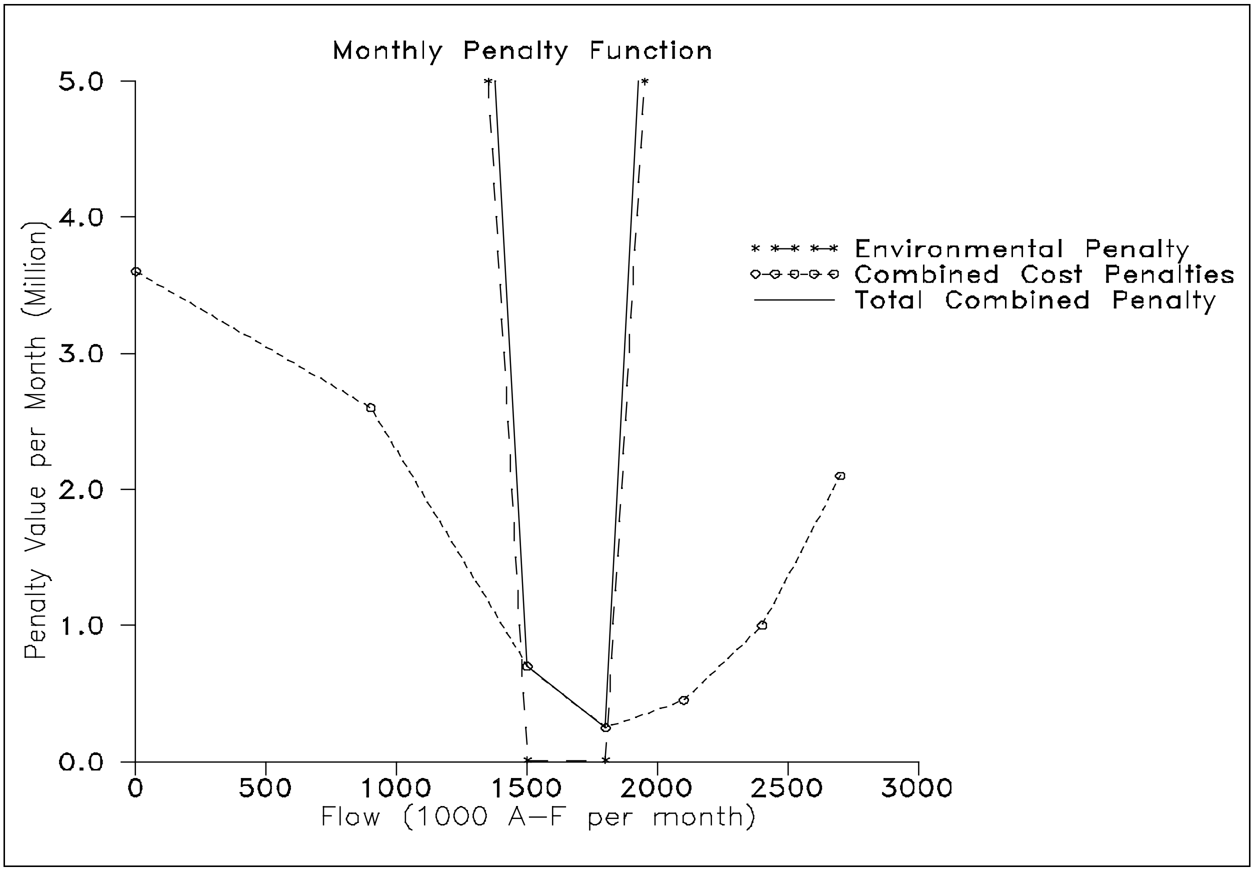

Figure B-17. Cost and non-cost based penalties combined

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B-7. Glossary

ARC. Connects two nodes of a network. In network-flow programming, each arc has three parameters: a lower bound, which is the minimal amount that can flow along the arc; an upper bound, which is the maximum amount that can flow along the arc; and a cost for each unit that flows along the arc.

CHANNEL-FLOW LINK. Represents the flow in a channel reach. A channel-flow link originates at any non-reservoir node and terminates at any network node.

CONSTRAINT. Limits the decision variables to their feasible or permissible values.

CONVEX FUNCTION. A function $f(X)$ for which the following is true for any two distinct points X_1 and X_2 and for $0<\lambda<1$: $f[\lambda X_1+(1-\lambda)X_2] < \lambda f(X_1)+(1-\lambda)f(X_2)$

DECISION VARIABLE. The unknowns which are to be determined from the solution of the model.

DIVERSION LINK. Carries flow out of the system. A diversion link originates at any system node and terminates at the sink node.

FINAL-STORAGE LINK. Carries flow out of the system, but only from a reservoir in the last period of analysis. It originates at a reservoir node and terminates at the sink node.

HYDROPOWER RESERVOIR-RELEASE LINK. Represents the release from a hydropower reservoir. The penalty function for a hydropower reservoir-release link depends on both the release from the reservoir and the storage in the reservoir.

INFLOW LINK. Brings flow into the reservoir-system network. An inflow link originates at the source node and terminates at any system node.

INITIAL-STORAGE LINK. Introduces to the network the volume of water initially stored in a system reservoir. The initial-storage link originates at the source node and terminates at a reservoir node in the first period of analysis only.

NETWORK. A collection of arcs and nodes.

NETWORK-FLOW PROGRAMMING. An optimization procedure for allocating flow along the arcs of a network. Network-flow programming is a special class of linear programming.

NODE. The junction of two or more network arcs. The node may represent a system reservoir, demand point, channel junction, diversion point. The sum of flow in arcs originating at a node equals the sum of flow in all arcs terminating at the node.

OBJECTIVE FUNCTION. Defines the overall effectiveness of a system as a mathematical function of its decision variables. The optimal solution to the model yields the best value of the objective function, while satisfying all constraints.

PENALTY FUNCTION. Defines the penalty for less-than-perfect operation as a function of flow, storage, or both.

PIECEWISE LINEAR APPROXIMATION. An approximation in which a non-linear function is represented by linear segments, arranged sequentially.

RESERVOIR-STORAGE LINK. Represents the volume of water stored in a reservoir at the end of a period. The link originates at any reservoir in a layered, multiple-period network and terminates at the node representing the same reservoir in the period following.

SIMPLE RESERVOIR-RELEASE LINK. Represents the total outflow from a non-hydropower reservoir. Flow in the link includes release and spill.

SINK NODE. The hypothetical absorber of all flow in the network. All diversion links and final-storage links terminate at the sink node.

SOLVER. Finds the minimum-cost allocation of flow to the network arcs, subject to the upper and lower bounds on arc flows and to continuity at the network nodes.

SOURCE NODE. The hypothetical provider of all flow in the network. All inflow links and initial-storage links originate at the source node. No user-defined links terminate at the source node.